Inter-municipal facility location-routing model

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Abstract

The planning of efficient and process-oriented inter-municipal street maintenance operations involve several decision making processes. Road maintenance for municipalities is an important activity to deliver value for money for the inhabitants and therefore has to be planned carefully. Nowadays Swiss municipalities have to deal with inefficient structures regarding the operational street maintenance in terms of cost, resources and labour which lead to high organizational costs. This paper is concerned with the planning of optimized multi-municipal facility locations in the field of operational street maintenance. The location of operation centres is one of the most important strategic decisions for municipalities performing inter-municipal street maintenance operations. Especially in times of declining public income, Swiss communities are trying to improve their efficiencies of operational street maintenance. In cooperation with the Swiss Federal Roads Authority (ASTRA), the Chair of Construction Process and Enterprise Management at the Institute of Construction and Infrastructure Management at ETH Zurich is developing an inter-municipal facility location-routing model in the field of operational road maintenance. It serves as a decision-making basis for street owners to improve the efficiency of operational road maintenance processes. This paper presents an inter-municipal facility location-routing and shows possible algorithms for optimizing global costs. Localization decisions present a very complex decision making problem that involve considerations of multiple criteria such as the minimization of build-up and installation cost of a new operation centre, transportation cost as well as an ideal availability to the municipal road network. The constructivist research approach will be applied to the development of the inter-municipal facility location-routing model. This research approach follows the hermeneutic research paradigm and aims to structure new socio-technical systems based on an intended input-output effect. The holistic facility location-routing model forms a basis for systematically developing operational street maintenance strategies. The aim is to ensure a high standard of street network quality, create a tool for local authorities, enabling them to cope more easily with challenging economic aspects and to provide good value to the customer and road users.

Keywords: operational street maintenance, facility location problem, cost optimization, inter-municipal cooperation, decision making tool for local authorities.

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1. Introduction

Nowadays Swiss municipalities have to deal with increasing financial pressure regarding the provision of public services. Increasing complexity and economic constraints, especially in the field of operational street maintenance, are the reasons why municipalities in Switzerland put more effort into cost savings and increasing efficiency, in order to ensure a high quality of communal infrastructure. In times where every municipality has its own operation centre with the corresponding equipment and team structure, inefficiencies in terms of cost, resources and labour can be observed. In cooperation with the Swiss Federal Roads Authority (ASTRA), the Institute of Construction and Infrastructure Management at ETH Zurich is developing an inter-municipal operational street maintenance model (I-OSMM). The aim is to motivate municipalities to collaborate regarding the execution of public tasks and to build inter-municipal structures to gain higher performance levels and to work more efficiently. Furthermore the equipment and infrastructure can be used more target-oriented and therefore organizational overhead costs savings can be obtained. This model is applicable where public services such as street maintenance operations, emergency services, fire departments, schools etc. need to be structured more efficiently to be able to cope more readily with challenging economic aspects. In the field of municipal street maintenance several research papers have been published which give direction to the following model (Girmscheid 2007a, Girmscheid 2007b, Girmscheid 2009, Fastrich and Girmscheid 2010a, Fastrich and Girmscheid 2010b). The Model serves as a basis in better decision making for inter-municipal cooperation, regarding the location of operation centres and optimizing cost and performance structures, with the aim of significantly improving efficiency in street maintenance, by using economies of scale.

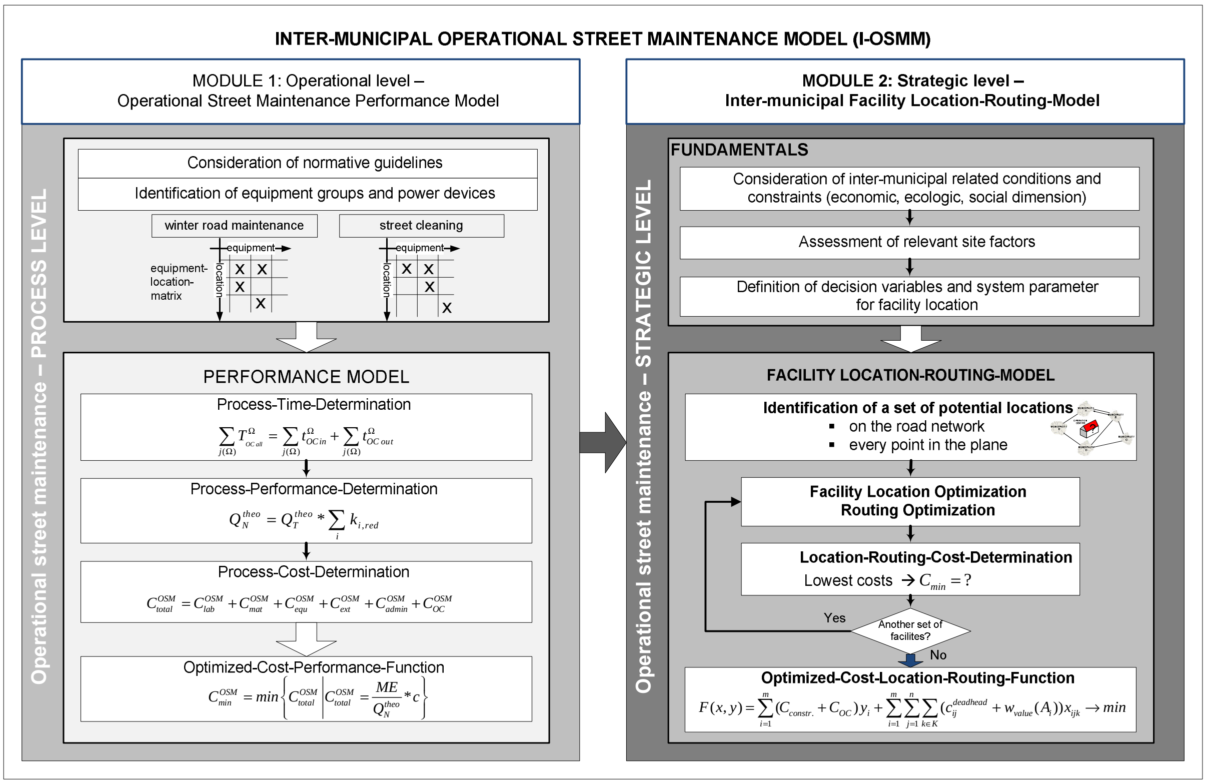


Figure 1: Overview – Inter-municipal operational street maintenance model

The I-OSMM consists of two modules – the operational street maintenance performance model on an operational level and the inter-municipal facility location-routing-model on a strategic level, as shown in fig. 1. Operational street maintenance contains, based on the Swiss Standard SN 640900a (2004), public tasks such as street cleaning operations, winter road maintenance and green area maintenance as well as the small rehabilitations of the street network (Girmscheid and Lindenmann 2008). To ensure a high level of street network quality, module one with its performance model of operational street maintenance shows a new approach of structuring and optimizing processes in the field of operational street maintenance, in order to increase the efficiency of public tasks in terms of scope, cost, resources and labour (Koller and Girmscheid 2012). This paper focuses on module two of an inter-municipal street maintenance model and shows how to locate operation centres when inter-municipal structures are used to optimize total cost. Furthermore, it provides a decision making tool for local authorities, enabling them to cope more easily with challenging economic and social aspects and to provide good value to the customer and road users.

2. Problem formulation and literature review

Road maintenance and asset preservation for municipalities is an important activity needed to deliver value for money for the residents. Decisions on where to locate facilities such as operation centres and satisfying the inhabitants' needs are a strategic issue for most Swiss municipalities and have a long-term effect on the operations of an inter-municipal street maintenance. The localization decision is a very complex decision problem that involves considerations of multiple criteria like the minimization of build-up and installation cost of a new operation centre, transportation cost as well as an ideal availability to the municipal road network.

Problems arising from the design and operation of street networks are especially complex. Facility location problems are optimization problems which follow the target of finding a subset of sites to establish a facility, by meeting all strategic and operational expectations of the service provider. Facility location problems and its variations have been widely studied in literature considering the field of operations research. Algorithms can be used according to the topography of the set of potential facilities as network locations models, models in the plane (Wesolowsky 1977) as well as discrete locations or mixed-integer programming models, respectively. Furthermore, objectives can be distinguished between minisum or minimax relations (Hakimi 1964, Klose and Drexl 2005). Minisum models are used to minimize the average distances of given networks, whereas minimax models are designed to minimize the maximum distance of the road network to the facility location. According to the type of work (winter road maintenance, street cleaning operations, etc.) in the field of operational street maintenance, the minisum model will be applied to gain a minimization of transportation costs between the demand points of the road network and the operation centre. While the classical location-allocation problem (LAP) only considers the optimized location of operation centres, the location-routing problem (LRP) also takes an optimal set of vehicle schedules and routing into account and therefore considers minimizing distribution costs (Salhi and Rand 1989). Several studies of LRP have been conducted, for instance, Golden (1977), Jacobsen (1980), Perl and Daskin (1985) have proposed LRP-algorithms with differing constraints, like capacity or tour-length limitations for example.

3. Research Methodology

The development of an inter-municipal facility location-routing-model of operational street maintenance is based on the constructivist research approach, which was developed by Glasersfeld (2008). The approaches' aim is to structure new socio-technical systems based on an intended input-output relation (Girmscheid 2004). The validation of the research project is based on triangulation. Triangulation ensures that the model is viable, valid and reliable. The modelling of the constructive-deductive process model is based on an intended input-output relation by using system theory (von Bertalanffy 1969). The viability will be ensured by using mathematical optimization methods like the graph theory and facility location problem. To ensure the reliability and validity of the model a realizability test will be conducted with eight municipalities of Switzerland to evaluate the current state empirically and to deduce the potential for efficiency improvement.

4. Inter-municipal facility location-routing-model

The purpose of the inter-municipal facility location-routing-model is to determine the optimized location of an operation centre, in order to satisfy the municipalities' needs and demands and to minimize the global costs incurred for each established facility. Furthermore it should motivate municipalities to collaborate and work together and therefore make it easier for them to cope with challenging economic constraints. With an inter-municipal cooperation, it strives to improve the efficiency regarding the cost and performance structures, by using economy of scales. The aim of an inter-municipal operation centre is to use resources (equipment, buildings, labour, etc.) more efficiently by benefiting from synergistic effects and know-how transfer, as well as the financial risk sharing regarding new expenditures and saving, due to optimized operation centre locations and vehicle routing. The model is especially applicable in questions where to locate facilities that are used collaboratively for the execution of public tasks. As shown in fig. 2 the first procedure starting the facility location-routing-process is the determination of inter-municipal related conditions and constraints regarding the economic, ecologic and social dimension.

Eiselt and Laporte (1995) show how to classify objectives in location models. These objectives can be manifold according to the municipalities needs, but mostly identified as the minimization of total construction, building and annual operation costs as well as the minimization of routing distances together with the maximization of the public service level provided for the municipalities. Recently, social and environmental aspects and objectives based on energy cost, pollution and noise, use of ecologically friendly de-icing techniques for example become increasingly important. Based on these constraints relevant site factors must be defined. After the assessment of site factors, decision variables and system parameters can be derived. According to Arinze and Banerjee (1992), possible system parameters are for example the topology of the municipalities, distance and travel time from the operation centre to the municipalities, number of possible facilities as well as the necessary capacity of the facility.

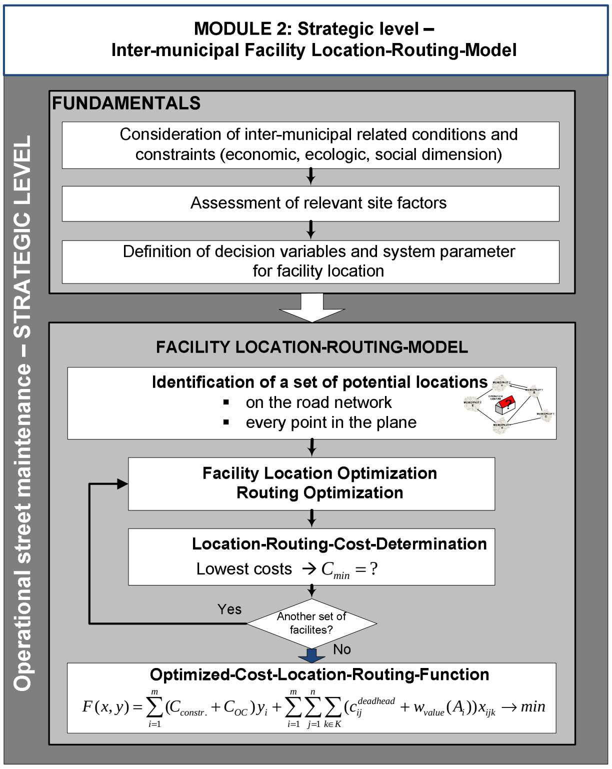


Figure 2: Inter-municipal facility location-routing-model

After defining the necessary decision variables and system parameters, potential operation centre locations must be defined. It should be noted that potential locations can be existing buildings which are suitable for the execution of tasks as well as new land where the new operation centre has to be build up. The inter-municipal facility location-routing-model therefore classifies the problem into two alternatives:

1.) **ALTERNATIVE 1:** Consideration of the facility location only
   a.) Formulation as warehouse-location problem (WLP) with one or more depots
   b.) Location in the plane with Euclidian metric

2.) **ALTERNATIVE 2:** Consideration of facility location and routing simultaneously
   a.) Location-Routing-Problem (LRP)

Fig. 3 shows possible alternatives (alternative 1a, 1b and alternative 2a) to locate an operation centre on a given street network or in the plane formed by five cooperating municipalities working together.

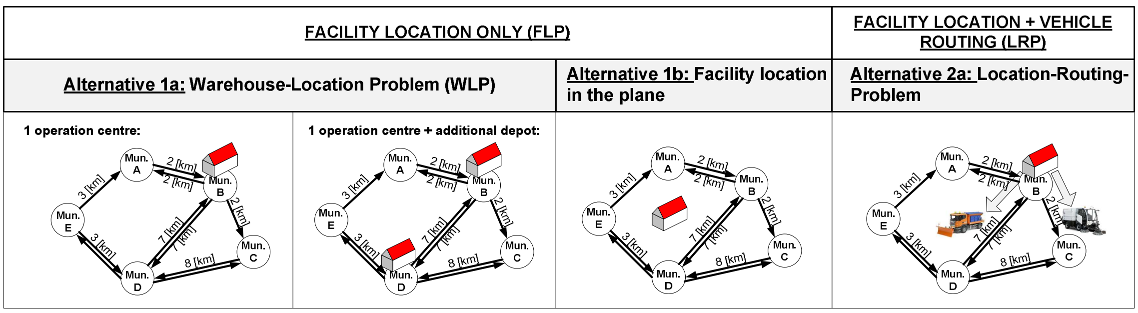


Figure 3: Alternatives for operation centre location and vehicle routing

The main difference of the proposed alternatives is the decision whether the operation centre will be located on a given street network or in the plane. These circumstances especially affect the cost structure, because of different requirements regarding new expenditures of establishing new roads and connecting supply ducts to the operation centre on the chosen "green field site". Alternative 2 also takes account of an optimal set of vehicle schedules and routing which isn't considered in alternative 1. In many cases in the field of operational street maintenance assumptions of linear costs regarding the warehouse location problem are not appropriate. Domschke (1996) for example, proposes to expect concave build-up costs for the operation centre. These costs are not constant, because usually they are dependent from the capacity of inventory. Therefore, doubling the capacity of space of the operation centre doesn't mean doubling the build-up costs. In the following, alternatives as proposed in fig. 3 will be discussed.

4.1 ALTERNATIVE 1: Facility location problem

Alternative 1 only considers facility location without vehicle routing. Alternative 1a can be formulated as a warehouse location problem (WLP). On a given street network one or more operation centres and depots can be established, depending on whether the total cost will be minimized by using one or more depots. Alternative 1b takes advantage of a "green field site", where every point in the plane can be seen as potential facility location.

4.1.1 Alternative 1a: Uncapacitated facility location problem (UFLP)

An uncapacitated facility location problem is a discrete problem, where a set of potential locations has to be allocated over a set of costumers with their specific demands and service requirements. The optimization problem is to find the subset of locations that will minimize the total fixed construction and operation costs and the variable transportation costs to satisfy the demand of public services in the municipalities. Considering the problem, let m be the number of potential operation centres, n the number of customers.

The decision variables for the optimization problem can be formulated as:

$$x_{ij} = \begin{cases} 1 & \text{If point i precedes point j on route} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{If the operation centre is located at site i;} \\ 0 & \text{otherwise} \end{cases}$$

The mathematical problem can be designed as:

$$F(x,y) = \sum_{i=1}^{m} (C_{Constr.} + C_{OC}) y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow min \qquad \text{(1-1a)}$$

$C_{Constr.}$ … construction costs of the building yard
C_{OC} … operation centre costs
c_{ij} … transportation costs for the execution of street maintenance operations

The objective in constraint (1-1a) is to minimize the total costs including the construction and operation centre costs as well as the variable transportation costs for the execution of street maintenance operations from the municipalities to the operation centre.

$$C_{OC} = C_{vh.maint.} + C_{coord.} + C_{operation} \qquad (2\text{-}1a)$$

Constraint (2-1a) shows that that the costs for the operation centre consist of costs for vehicle maintenance, cost for coordination and organization (overhead costs) and operational costs of the building itself.

$$\sum_{i=1}^{m} x_{ij} = 1; \qquad j = 1,...,n \qquad (3\text{-}1a)$$

Constraint (3-1a) ensures that each municipality is served by only one operation centre.

$$x_{ij} \le y_i; \qquad i = 1,...,m; \quad j = 1,...,n \qquad (4\text{-}1a)$$

To ensure that the demand requirements of the municipalities will be met, constraint (4-1a) is invented. It ensures that the operation centre can satisfy the demand of public services (such as winter road maintenance, street cleaning operations etc.) of the municipalities.

$$\begin{matrix} y_i \in \{0,1\} & i = 1,...,m; \\ x_{ij} \ge 0 & \forall i,j \end{matrix} \qquad (5\text{-}1a)$$

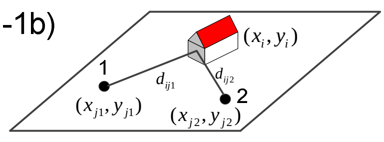
In constraint (5-1a) the 0/1-decision variables regarding the operation centre will be defined.

4.1.2 Alternative 1b: Continuous location model

Models in the plane are characterized by the fact that it is feasible to locate an operation centre at any point in the plane. The distance between two points can be measured by suitable metrics, in order to minimize the sum of distances between the operation centre and the points of demand. As Klose (2005) stated the subject of the so called Weber problem is to determine the coordinates (x, y) ∈ $R \times R$ of a single facility such that the sum of the distances to given demand points will be minimized.

The optimization problem can be formulated as:

$$F(x,y) = \sum_{i=1}^{m} (C_{dev.} + C_{constr.} + C_{OC}) y_i + \sum_{j=1}^{n} d_{ij}^2 x_{ij} \rightarrow min \qquad (1\text{-}1b)$$



$C_{dev.}$ … development costs regarding the new operation centre

d_{ij}^2 … Euclidean distance metric [km]

The target function (1-1b) aims to minimize the cost for development, construction and operation of the building yard together with the minimization of the Euclidean distance from every possible point in the plane to the new operation centre.

$$C_{dev.} = C_{road\ constr.} + C_{land} + C_{connection} \qquad (2\text{-}1b)$$

As shown in constraint (2-1b) the costs for development of the operation centre consist of the costs for the establishment of new connecting roads to the existing road network, costs for land use as well as connection costs for electricity, drains, water etc.

$$d_{ij}^2 = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \qquad (3\text{-}1b)$$

Constraint (3-1b) shows the Euclidean distance from every possible point in the plane $R \times R$ to the new operation centre which needs to be minimized to reduce transportation costs.

4.2 ALTERNATIVE 2: Facility location problem and routing

Location-routing problems (LRP) consist of two linked subproblems: the facility location problem and the vehicle routing problem. Fig. 4 shows a flowchart of the heuristic design of the LRP for inter-municipal street maintenance (adapted from Lin and Kwok (2006); Tavakkoli-Moghaddam, Makui et al. (2010)). The two subproblems – facility location and routing – are tackled repeatedly for a predefined minimum number of facilities, until the total cost reach the optimum.

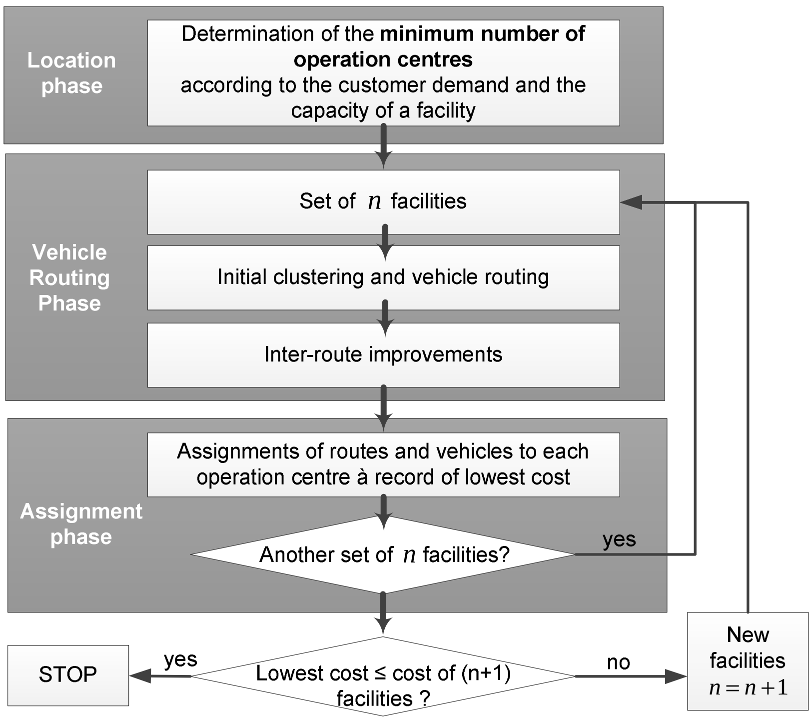


Figure 4: Heuristic design for the operation centre location-routing-problem (LRP) (Lin 2006; Tavakkoli-Moghaddam, Makui et al. 2010)

The first step – the location phase – starts with defining a minimum number of potential facilities, in order to fulfil the municipalities work requirements. For every set of potential sites the vehicle routing process will be applied. With a given set of potential operation centres, an initial clustering and vehicle routing will be conducted. This can be done by either using the saving algorithm by Clarke and Wright (1964) or the traveling salesman algorithm (TSP). In the routing phase Clarke Wright's saving algorithm will be applied for optimizing each objective function with the aim to achieve local optima. Inter-route improvements can be solved by applying the TSP. In the last assignment phase assignments of routes and vehicles to each operation centre or depot will be done in order to record the lowest cost. The approach will be terminated when the set-up cost with an additional facility exceeds the lowest total cost calculated.

The mathematical problem can be designed as:

$$F(x,y) = \sum_{i=1}^{m} (C_{constr.} + C_{OC}) y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k \in K} (c_{ij}^{deadhead} + w_{value}(A_i)) x_{ijk} \rightarrow min \qquad (1\text{-}2a)$$

$c_{ij}^{deadhead}$ … deadhead costs to and from the operation centre

$w_{value}(A_i)$ … variable costs for value-adding work in relation to the worked area

The aim of constraint (1-2a) is to minimize the total costs including the construction, building and operation centre costs (containing costs for vehicle maintenance, management costs and operational costs). The transportation costs can be divided into deadhead costs to and from the operation centre which can be seen as fixed costs as well as variable costs for value-adding tasks (such as winter road maintenance, street cleaning operations,…) in relation to the worked area in the municipality.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = 1; \qquad i = 1,\dots,m \qquad j = 1,\dots,n \qquad (2\text{-}2a)$$

Constraint (2-2a) ensures that each municipality is served by only one operation centre and each vehicle is assigned to only one route.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} b_j(S) x_{ij} \le V_C; \qquad \forall k \in K \qquad (3\text{-}2a)$$

$b_j(S)$ … demand of public service required in the municipality in relation to the required service level

V_C … vehicle capacity; e.g. loading volume, mechanical dimensions (width of snow plough, etc.)

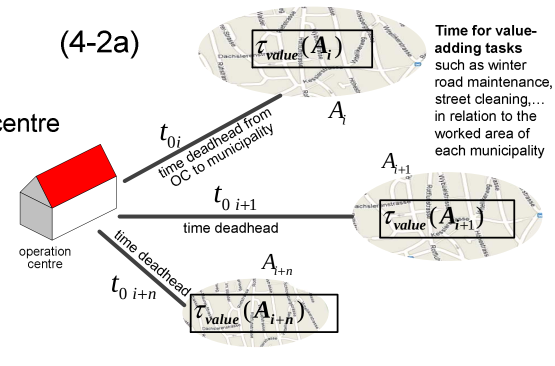
Constraint (3-2a) takes account of a technical vehicle influencing factor V_c, in order not to exceed the demand of public service in relation to the required service level (S) in the municipality.

$$\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \tau_{value}(A_i) \le T_K; \quad \forall k \in K \qquad (4\text{-}2a)$$

t_{ij} … deadhead time to and from the operation centre

τ_{value} … time for value-adding tasks

$A_{i,\dots,i+n}$ … worked area of municipality i,…,i+n



Constraint (4-2a) guarantees that each vehicle route doesn't exceed the maximum allowable duration T_k according to the legal working hours per day. Variable t_{ij} shows the time of deadhead (non-value-adding) from the operation centre to the site of operation. With τ_{value} the time for value-adding tasks can be described in relation to the worked area in the municipality.

5. Results

Results that can be obtained by using the inter-municipal facility location-routing model are schematically shown in fig. 5. The result of the optimization process regarding the facility location and vehicle routing show different cost-functions $C_{total}^{(1a,1b,2a)}$ in relation to the facility location, using alternatives 1a, 1b or 2a. The result is an optimized cost-location-routing-function which shows (under economic and operational constraints) where to locate new facilities. A prerequisite for this is the establishment of an inter-municipal cooperation. Variable $C_{fix}^{(1a,1b,2a)}$ denotes the amount of total fixed cost of the operation centre, regarding the land cost, development cost, construction and building costs of the operation centre as well as operation costs. Every inter-municipal merger has different demands in public services in relation to the required service level. Therefore, $y^{(1-3)}$ will be integrated in the model to show the overall demand of public service. The last step will be the calculation of an envelope (shown as thick line in fig. 5) which shows optimal solutions by connecting fixed costs of the operation centre and depots and demands of every cost function on the envelope (Schumann, Meyer et al. 2011). Therefore, for every demand y, an optimal solution regarding minimum costs can be found on the envelope.

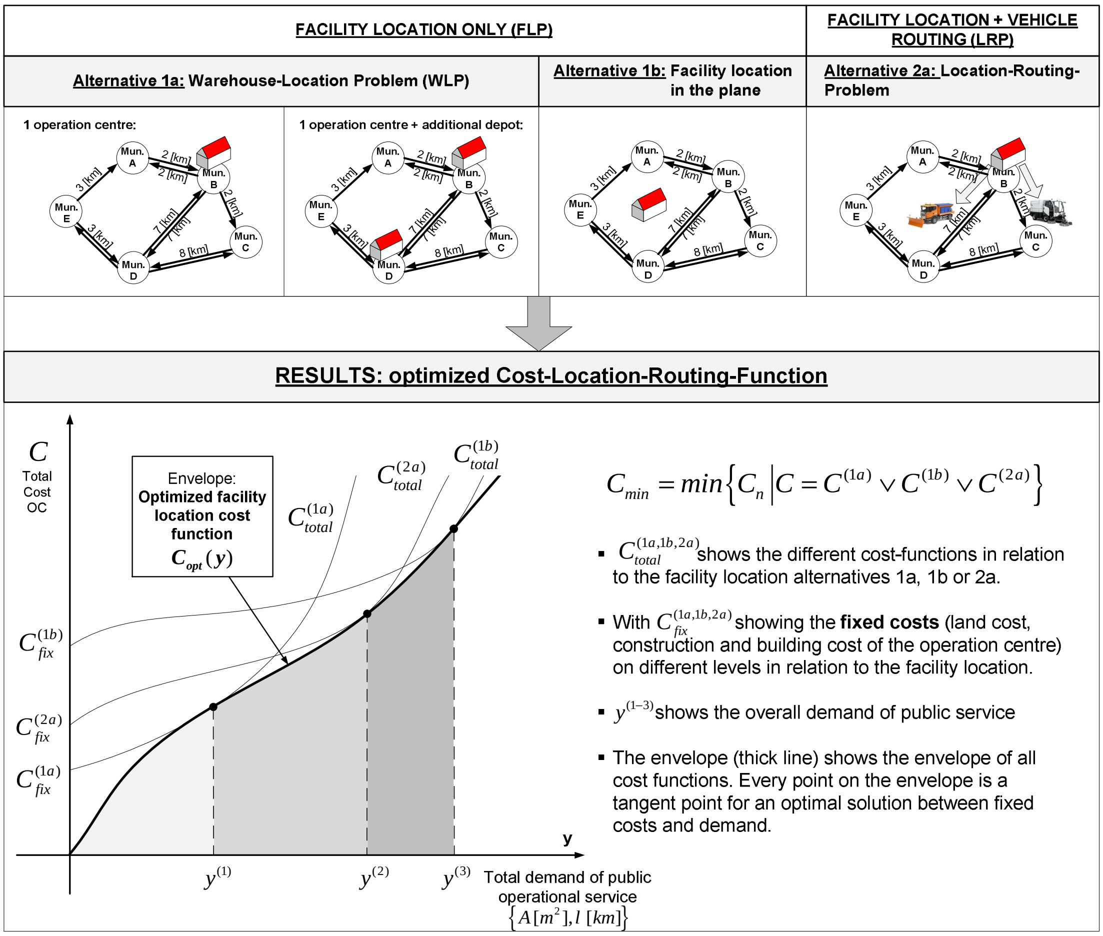


Figure 5: Results – Optimized Cost-Location-Routing-Function

6. Conclusion and outlook

This paper focuses on module two – the inter-municipal facility location-routing-model - of a holistic model of an inter-municipal street maintenance within the framework of the development of a research project on behalf of the Swiss Federal Roads Authority. The aim of this paper is to set-up an inter-municipal facility location-routing model in the field of operational street maintenance, aiming to make a valuable contribution to the process-oriented execution of public tasks. It shows how to locate operation centres, in order to minimize global cost by providing a high-quality inter-municipal street network and to use equipment and team structures more efficiently. Furthermore, the inter-municipal facility location-routing model provides a decision making tool for local authorities, to be able to cope more readily with challenging economic aspects and to provide good value to the customer and road users. The subsequent step in the development of the model will be the calculation and comparison of the proposed facility location and vehicle routing alternatives, so to provide recommendations for local authorities and therefore to increase the efficiency of public inter-municipal tasks.

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